Welcome to Geometry!

Geometry is one of the oldest areas of mathematics that deals with properties of space. We'll be learning about shape, distance, size, and position of objects.

We will discover many important relationships using the same tools that ancient mathematicians used many years ago: the *straight edge* and the *compass.*

The Point

A point is a location represented by a dot. It does not have any length, thickness, shape or size, it only has a position.
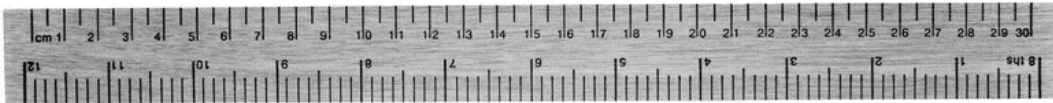
•

The Line

A line is straight, goes on forever, and has no thickness or depth.

⟵————————————⟶

The Straight Edge is also called a *ruler* or a *straight tool*. We use these to draw straight lines or to check a line for straightness.
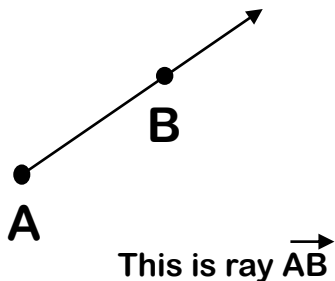
How many lines can you draw that touch the points below?
How many points does each line touch?

•

•

•

•

The Ray

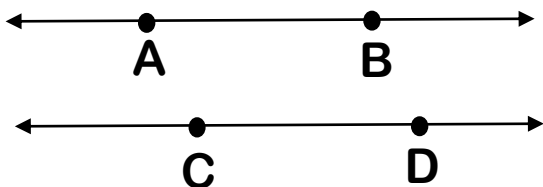**The ray is like a laser – it starts at one end and goes on infinitely.**

B

A

This is ray $\overrightarrow{AB}$

The Line Segment

The line segment starts at one end and ends at the other.

A　　　　　**B**

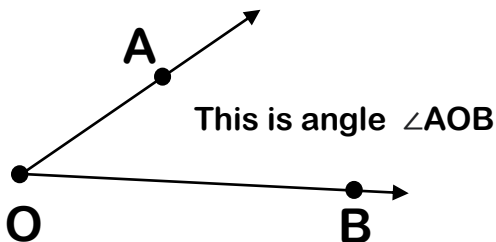**Draw a line that connects points A and B to make a line segment $\overline{AB}$**

Parallel lines never meet.

A　　　**B**

C　　　**D**

Line AB is parallel to Line CD: $\overleftrightarrow{AB}$ || $\overleftrightarrow{CD}$
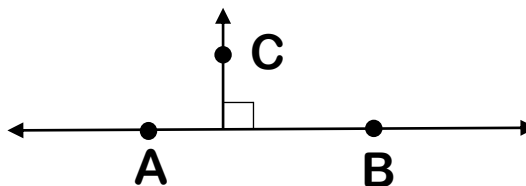
Draw two sets of parallel lines here:

The Angle

An angle is when two rays meet at a point, measured in "degrees".

A

This is angle ∠AOB

O　　　　**B**

Draw angle CDE here:

C

E

D
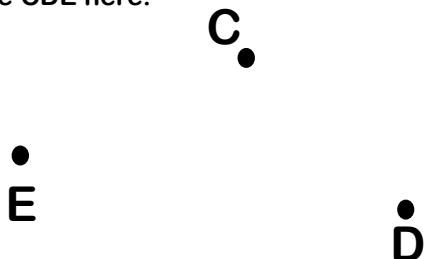
Perpendicular lines meet at a 90 degree angle.
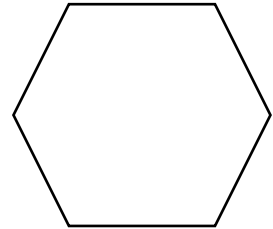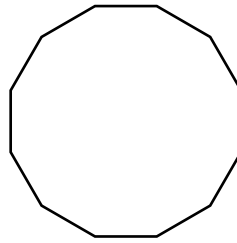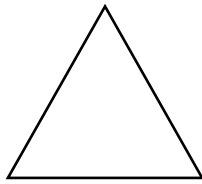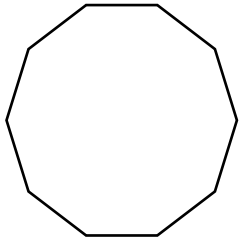
C

A　　　**B**

Line AB is perpendicular to Line CD: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

Draw two perpendicular lines here:

Basic Geometry Shapes
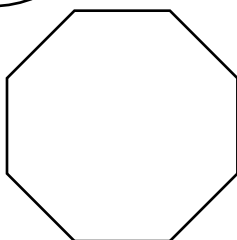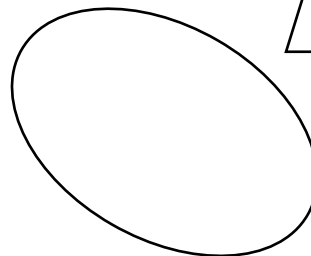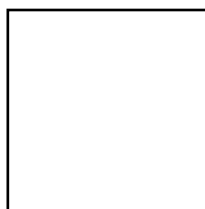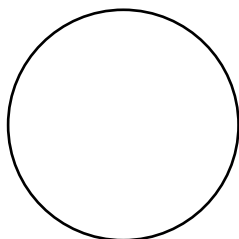
Three sides, four sides, five, six, seven, eight… how many do you know?

Write the number inside the corresponding shape:

1. **Circle**
2. **Square**
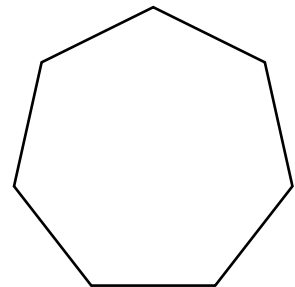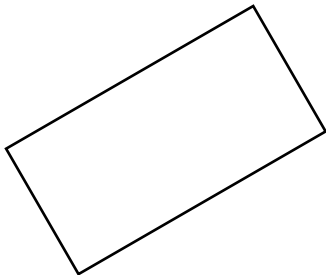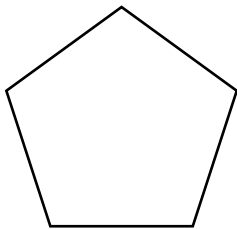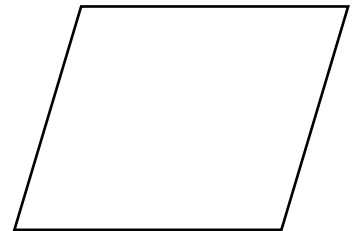3. **Triangle**
4. **Rectangle**
5. **Pentagon (5 sides)**
6. **Hexagon (6 sides)**
7. **Heptagon (7 sides)**
8. **Octagon (8 sides)**
9. **Parallelogram**
10. **Decagon (10 sides)**
11. **Ellipse**
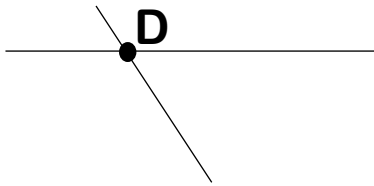12. **Dodecagon (12 sides)**
13. **Trapezoid**
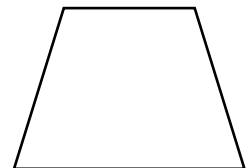
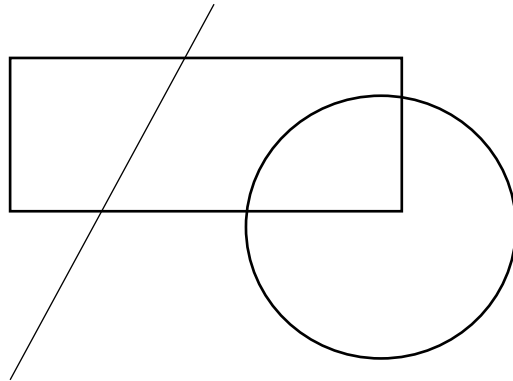**A polygon is a 2D closed figure with straight sides.**

A quadrilateral is a 2D shape with four sides, four vertices and four angles.

Point of Intersection
This is a point where two line segments meet or two lines meet.

Label all the points of intersection with different letters:

D

Draw all the line segments you can that connect A, B, C, D, E, F, G and O.

A

B

C

G

O

E

D

How many line segments did you draw?_____

How many intersections?_____

How many triangles? _____

How many quadrilaterals? _____

Are there any other shapes? _____

A figure with six sides is a *hexagon.*

Intersections
This is a point where two lines or curves meet.

Draw $\overline{AB}$ and $\overline{DE}$. Label the intersection C.
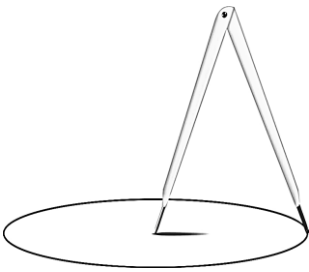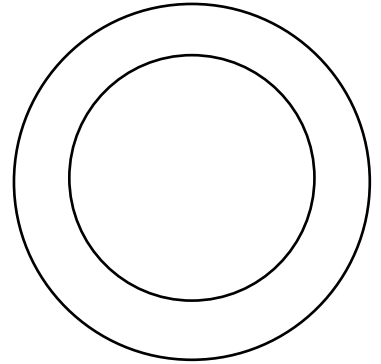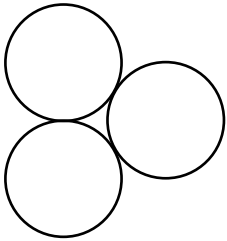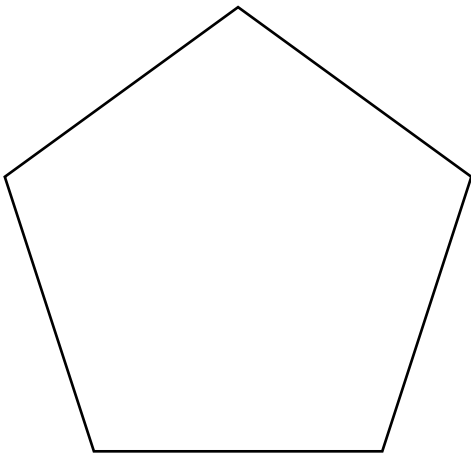
•
E

A•

•**B**

•
D

Draw a triangle, a quadrilateral, and a line so that they create six intersections. Label each intersection with different letters.

The Circle
**The circle is a round figure with no corners, no edges, and no flat lines.
Use your compass to make different circles and shapes!**

Use your compass to draw a circle <u>around</u> the pentagon:

Use your compass to draw a two circles inside another circle:

Draw a circle with the center at A, includes point B *on* the circle, with point C *outside* the circle.

A •   • B   • C

Now draw a circle with the center on C which has the point B outside of it.

Draw a circle with center B that intersects both of the first two circles.

The Radius
This is measured *from* the center of a circle to any point *on* the circle.

Draw a circle with the center on Z.

Draw a line segment from Z to the circle. This line segment is your *radius.*

Z •

Measure the radius with your ruler:

_____ **cm**

Now extend your line segment so it also touches the circle on the other side. This is your *diameter.*

Measure the diameter with your ruler:

The Diameter
Any straight line passing through the *center* of the circle with endpoints *on* the circle.

_____ **cm**

Measuring Radius and Diameter
Use your ruler and measure the radius and diameter of each circle!

Draw three circles of different sizes at the points below.

A •

• B

• C

**Draw a line through the center of each circle
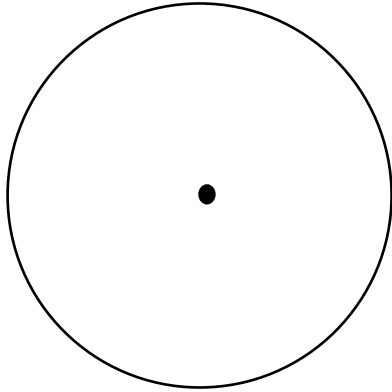with your ruler, intersecting the circle twice.**

**Measure both the *diameter* and the *radius*
of each circle (using inches or cm)
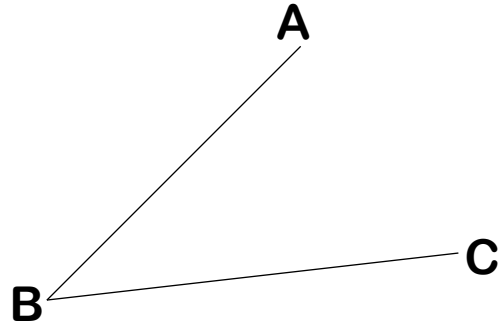and write these next to each circle.**

Congruent
Congruent means to match *exactly* in both size and shape.

Draw three radii in the circle below.

Use your compass to answer:

A

C

B

Are these radii congruent?_____

Are AB and BC congruent?_____

Draw a circle with center A and radius AB.

Now draw a circle with center X with a radius congruent to AB.

Choose a point on the circle X and label this point as "Y". Draw XY.

Are the two lines congruent?_____

A

X •

B

Congruent
Congruent means to match *exactly* in both size and shape.

Draw two congruent circles.	**Use your compass to answer:**
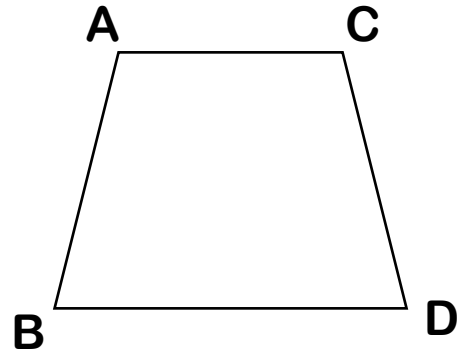
Are the radii congruent?_____   **Are $\overline{AB}$ and $\overline{CD}$ congruent?**_____
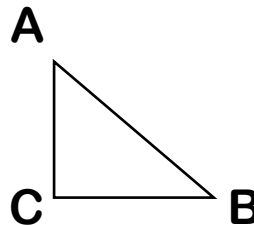
Draw a circle with center A and radius $\overline{AB}$.

Now draw a circle with center X with a radius congruent to $\overline{AB}$.

Choose a point on the circle X and label this point as "Y". Draw $\overline{XY}$.

Are the two lines congruent?_____

X
•

A

C   B

Measuring Angles with a Protractor
Angles are measured with a protractor. Angles are measured in *degrees.*

We measure line segments with a ruler.

When we have an angle, we measure how "open" the angle is.

This line is "40 degrees", which we write as **40º**.
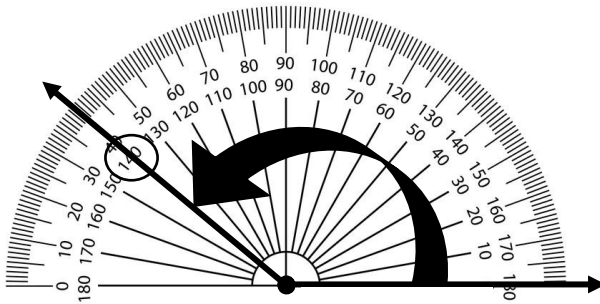
Notice that we are reading the top scale.

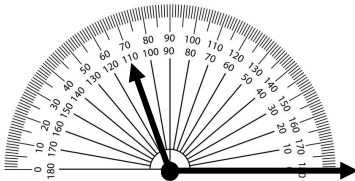Always start at the zero and use the scale that matches with where you started.
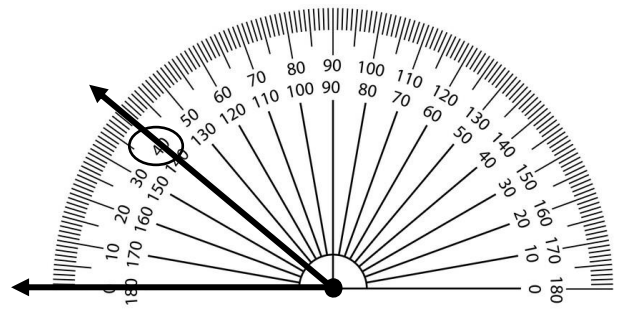
This time, we read the bottom scale because the zero is on the right.

We read numbers increasing from zero on the lower scale.

This angle is now "140 degrees", which we write as **140º**.

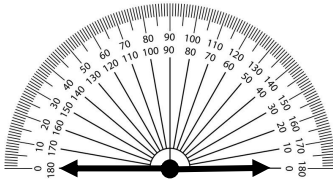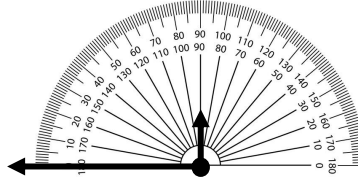## Measure the angle by extending the line through the scale:
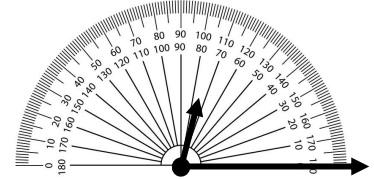
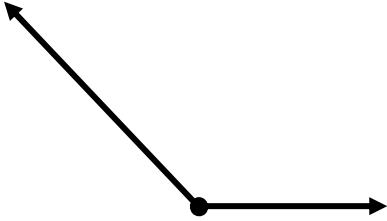**Angle:**
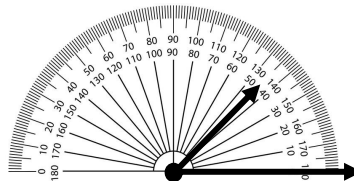
Angle:
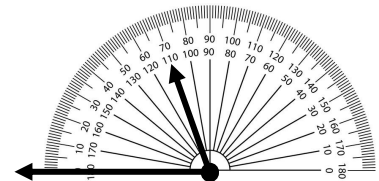
Angle:
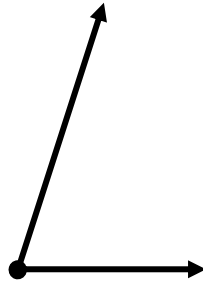
Angle:

Angle:

Angle:

Measuring Angles with a Protractor
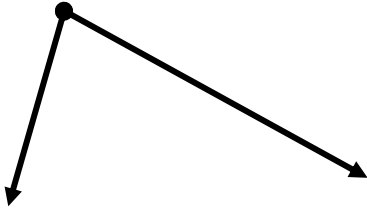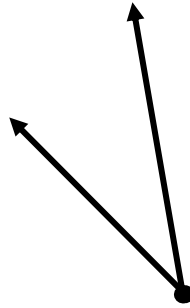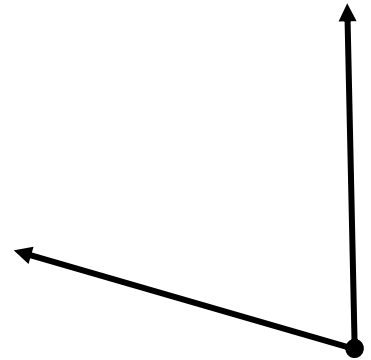### Measure each angle using your own protractor.
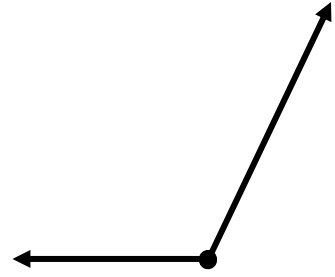
Angle:	Angle:	Angle:
Angle:	Angle:	Angle:

You can extend the lines first if they are too short to read on your protractor.
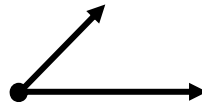
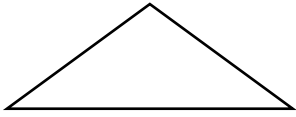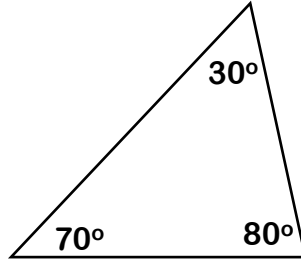|		
Angle:	Angle:	Angle:
Angle:	Angle:	Angle:

Special Properties of Triangles
Triangles have properties we can use to find out everything about them!

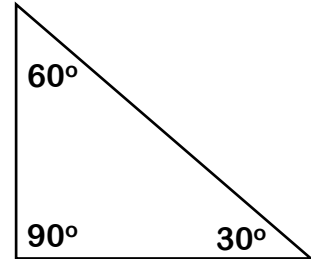**A triangle has three sides and three angles.**

Obtuse triangles have one angle greater than 90º.
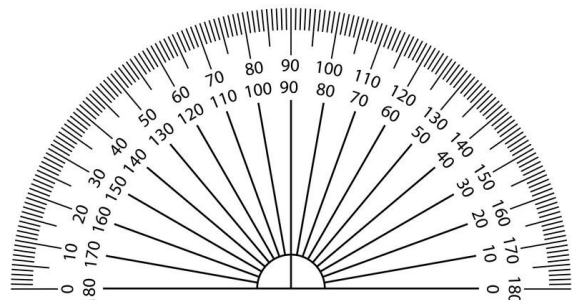
The sum of the angles always add up to 180 degrees.

30º

70º   80º

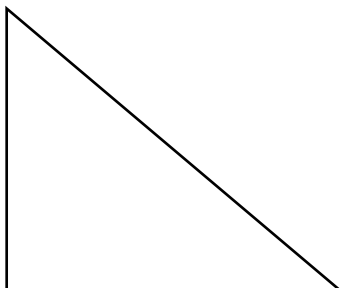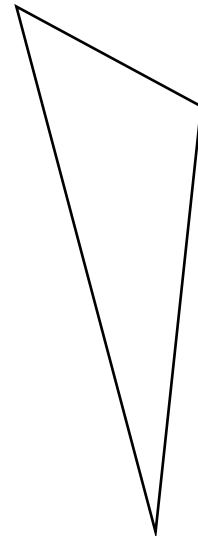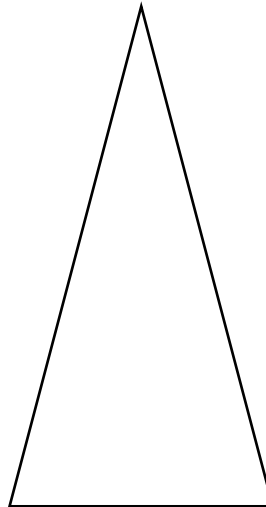**Acute triangles have all angles less than 90º.**

A right triangle has one angle equal to 90 degrees.
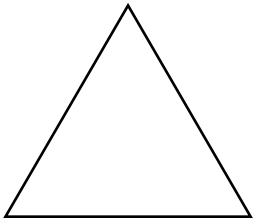
60º

90º   30º

**Use your protractor to measure the angles of each triangle.
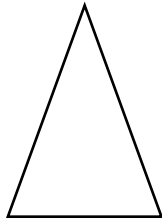Label the type of triangle as acute, right or obtuse.**
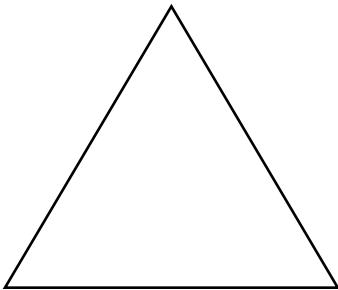
Special Triangles

These triangles show up often enough that they have their own special properties!

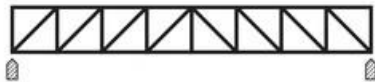| An equilateral triangle has three equal sides and three equal angles.	An isosceles triangle has two equal sides and two equal angles.	A scalene triangle has no equal sides and no equal angles. Draw three examples here:

These are steel truss roof frame designs.
Which types of triangles do you notice in each?

HOWE TRUSS
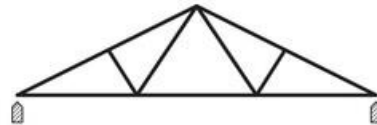
FINK TRUSS

PRATT TRUSS

SCISSORS TRUSS

WARREN TRUSS
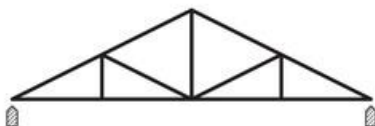
KING POST TRUSS
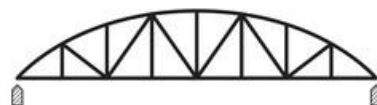
BOWSTRING TRUSS

Pick your favorite design. How many triangles do you notice? _____

How many of the following are in your favorite truss?

Equilateral: **Isosceles:**   **Scalene:**

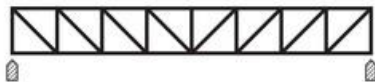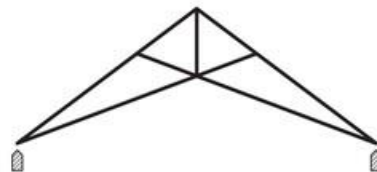# Measuring Interior Angles
Interior means inside, exterior means outside.

Measure the interior angles:

How many angles are the same?

Measure the interior angles:

How many angles are the same?

Measure the interior angles:

How many angles are the same?

Measure the interior angles:
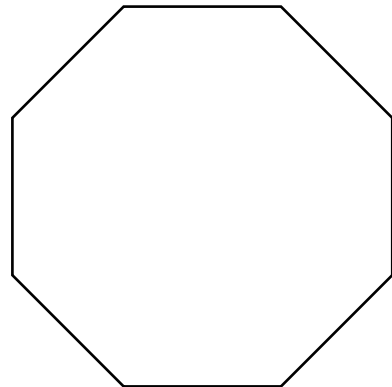
How many angles are the same?

Right Angles

Right angles are made from a line that is perpendicular to a straight line.

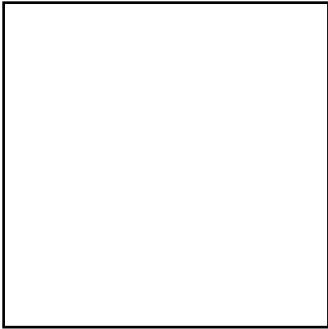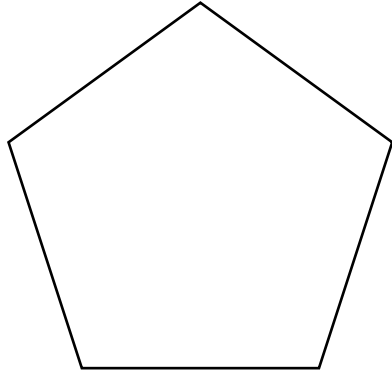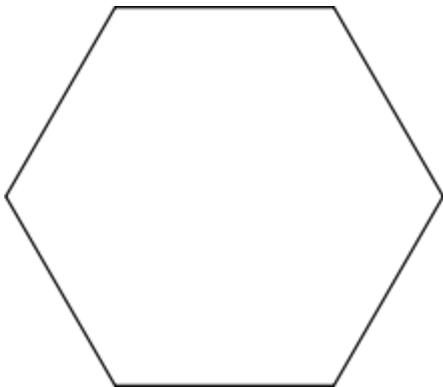**Draw a line perpendicular to $\overline{EF}$ at point G:**

D

E **G** **F**

Draw a line perpendicular to $\overline{ABC}$ at point B:

A ———————————————————————— **C**
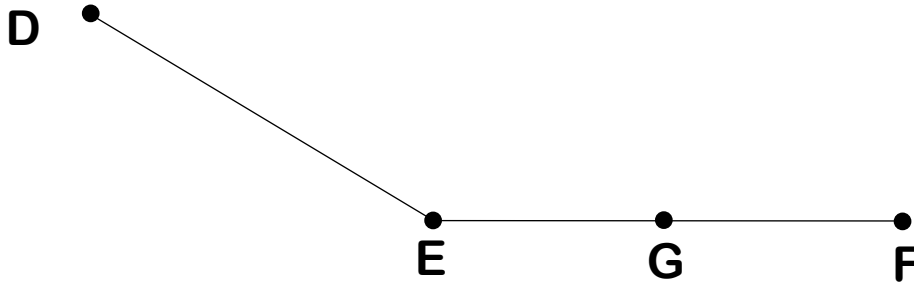
B
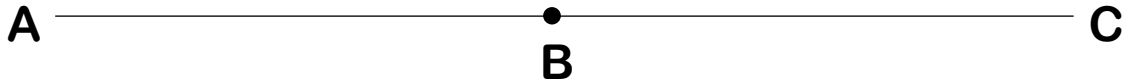
Pick a point on the perpendicular line you drew above and add point D. ∠ABD is a *right angle.* A right angle is always half of a straight angle.

Measure ∠ABD:

Draw a perpendicular to the line below through point B and label with a perpendicular symbol:

B
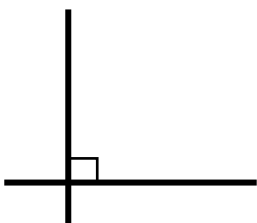
Perpendicular Symbol

All About Squares

A square is a quadrilateral with four equal sides and four right angles.

A   **B**

ABCD is a square.

Compare the sides of the square. Are they all congruent?

Compare all of the angles. What do you notice?

D   **C**
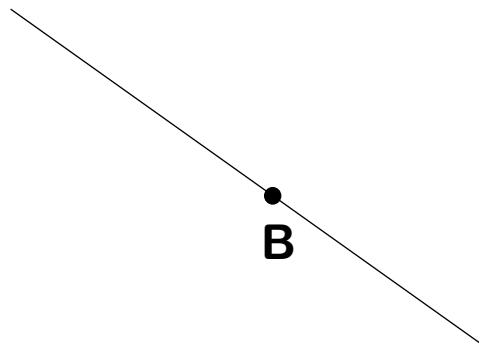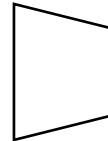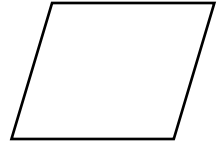
Which of the following quadrilaterals are squares? (circle):

Let's construct a square!

1. **Use your protractor to mark two points, one above A and the other above point B. Line up the protractor's hole at A and make a mark at 90º. Do the same to mark a point above point B.**

2. **Draw a line perpendicular to A through the point above it. Do the same for B.**

3. **Measure line AB.**

4. **Use this measurement to determine how high to make the sides of the square. Mark the endpoints with C and D.**

5. **Draw CD.**
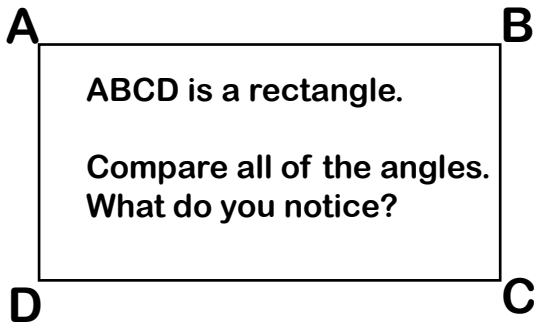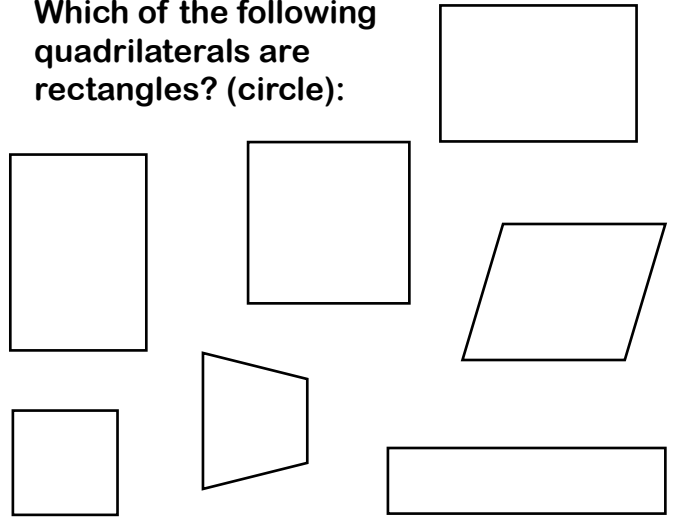
A   **B**

Check:
- are all four sides congruent? (use a ruler!)
- are all four angles right angles? (use a protractor!)

All About Rectangles

A rectangle is a quadrilateral with all right angles.

A   **B**

ABCD is a rectangle.

Compare all of the angles.
What do you notice?

D   **C**

Which of the following
quadrilaterals are
rectangles? (circle):

Let's construct a rectangle!

1. Draw perpendiculars to segment $\overline{PQ}$ through points P and Q.
2. Choose a point on the perpendicular through P and label it "A".
3. Measure $\overline{PA}$.
4. Use the line length $\overline{PA}$ to mark "B" on the perpendicular line through point Q.
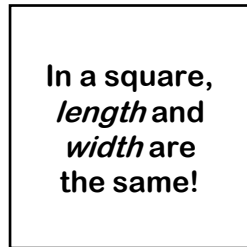5. Draw $\overline{AB}$.

P   **Q**

Check:
 - are all four angles right angles? (use your protractor!)
 - how do you know this object is a rectangle?

Area of Rectangles and Squares
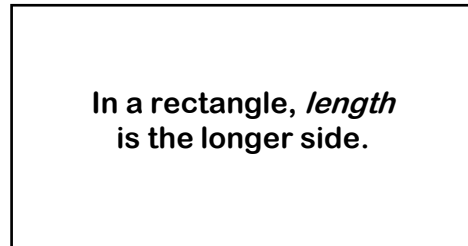## Area is the size of a surface of a two-dimensional figure.

Think of area as the *square units* that a shape covers.

In a square, *length* and *width* are the same!

width

length

Area is the *length* times the *width* of a rectangle and a square.

In a rectangle, *length* is the longer side.

width

length

Area is measured in *square units*:
- **square inches (in^2)**
- **square feet (ft^2)**
- **square cm (cm^2)**
- **square meters (m^2)**
- **square miles (mi^2)**
- **square kilometers (km^2)**

Area = L x W

Area = 4 x 2 = 8 in^2
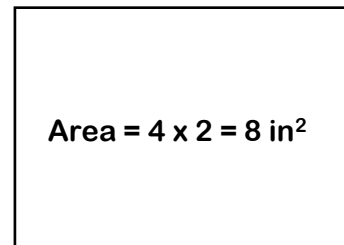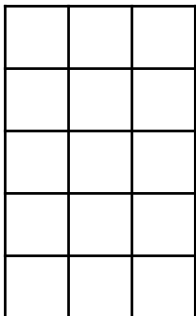
width = 2 inches

length = 4 inches

Count the number of squares to find the dimensions and area of each rectangle:
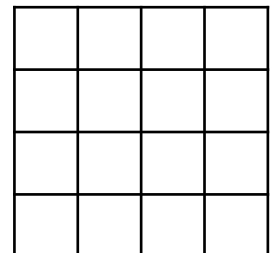
Length:

Width:

Area:
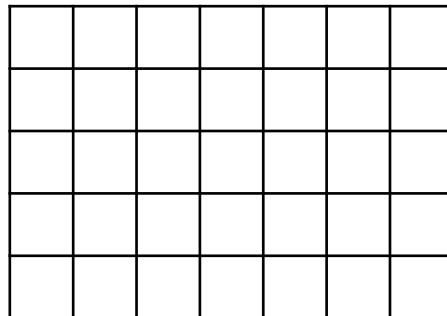
Length:

Width:

Area:

Length:

Width:

Area:

Area of Rectangles and Squares

Find the area of each shape using Area = Length x Width
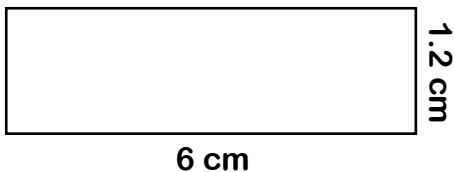
3 cm (left side)
3 cm (bottom)

Don't forget units in your numbers!

1.2 cm
6 cm

4 inches
9 inches

Length:

Width:

Area:

Length:

Width:

Area:

Length:

Width:

Area:

32 feet
75 feet

0.75 miles
2.1 miles

300 km
300 km
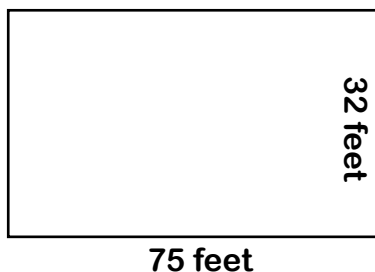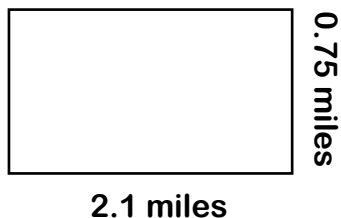
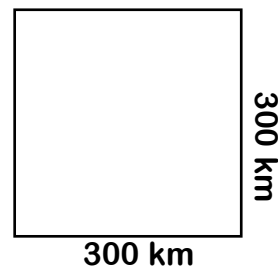**Length:**

Width:

Area:

Length:
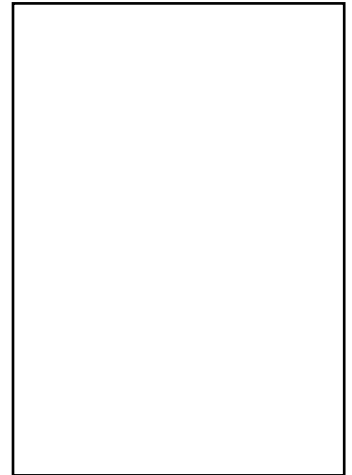
Width:

Area:

Length:

Width:

Area:

Area of Rectangles and Squares

Use your ruler to measure each shape and find the area using Area = Length x Width

Measure shapes in inches or cm to the nearest tenth

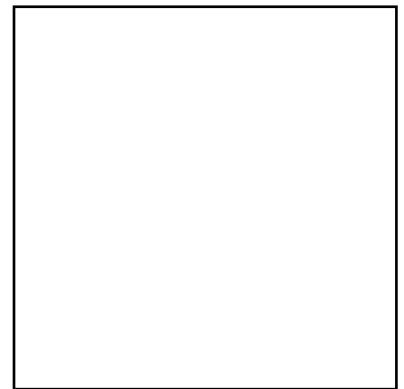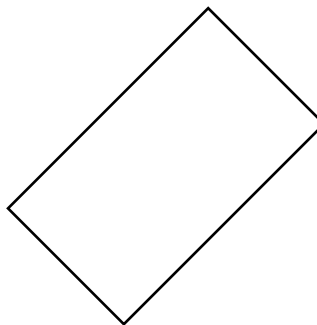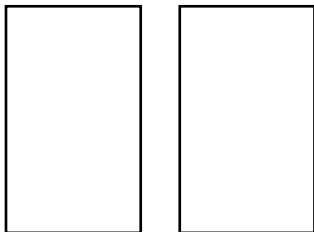**Don't forget units in your numbers!**

Length:

Width:

Area:

Length:

Width:

Area:

Length:

Width:

Area:

Length:

Width:

Total Area:

Length:

Width:

Area:

Length:

Width:

Area:

Area of Rectangles and Squares

Using Area = Length x Width find the missing dimension(s) for each shape.

Area of this square is 25 cm^2

Don't forget units in your numbers!

Area is 25 cm^2

10 ¼ cm

6 ½ inches

Area = 64 in^2
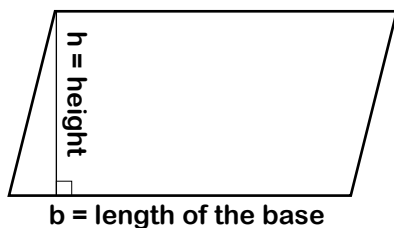
Area = 89 ft^2

36 feet

Area = 3.5 ft^2

18 inches

Area of square is 6.4 mi^2
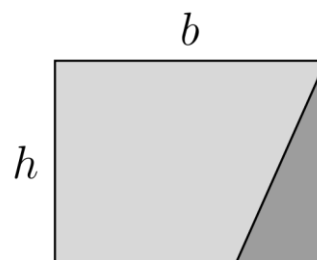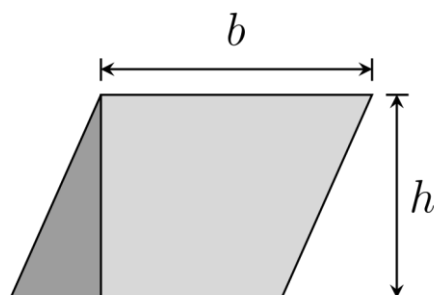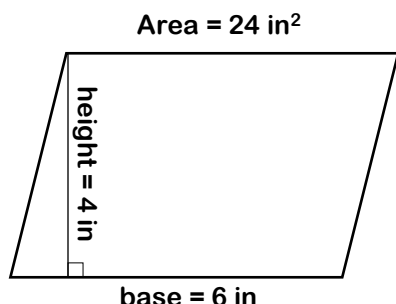
1 foot = 12 inches

Area of a Parallelogram

A parallelogram is a simple quadrilateral with two sets of parallel sides.

h = height

b = length of the base

Area for a parallelogram is the height times the length of the base.

Make sure the height makes a 90° angle with the base.

Area = b x h

Area = 24 in^2

height = 4 in

base = 6 in

b

h

b

h

A parallelogram can be rearranged into a rectangle with the same area.

Find the dimensions and area of each parallelogram:
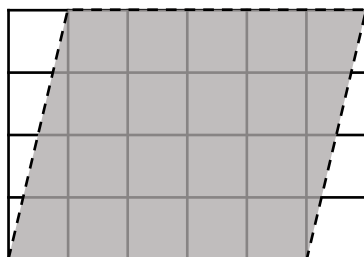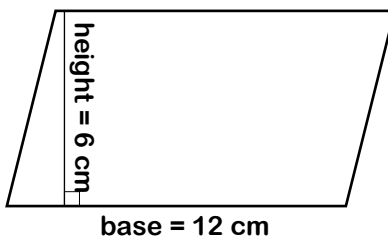
height = 6 cm	height = 3 in	
base = 12 cm	base = 5 in	

Base:

Height:

Area:

Base:

Height:

Area:

Base:

Height:

Area:

Area of Parallelograms

Find the area of each shape using Area = base x height

Don't forget units in your numbers!

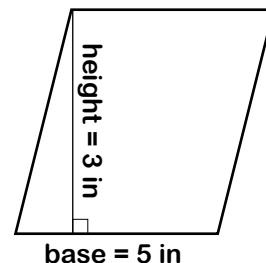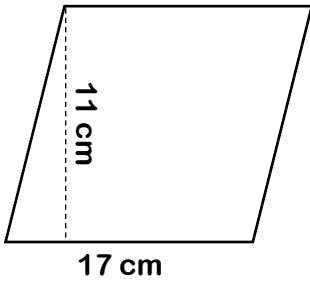|		
11 cm 17 cm	1.2 feet 0.9 feet	1.3 in 2.6 in
Base: Height: Area:	Base: Height: Area:	Base: Height: Area:
4.6 km 1.1 km	0.3 miles 1.5 miles	25 mm 20 mm
Base: Height: Area:	Base: Height: Area:	Base: Height: Area:

Area of Parallelograms
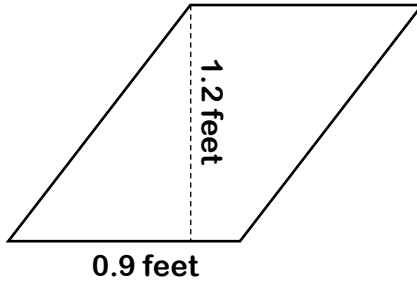
Find the area of each shape using Area = base x height
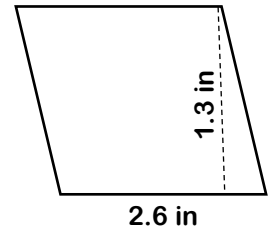
Don't forget units in your numbers!

Area = 132 cm²

11 cm

Area = 1.6 ft²

0.8 feet

Area = 7.28 in²

2.6 in
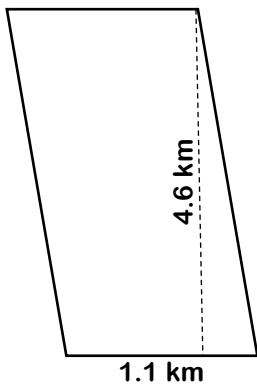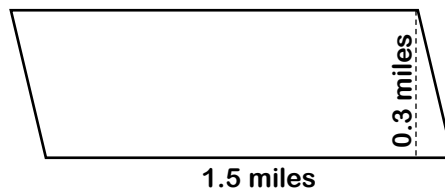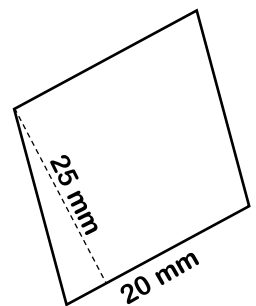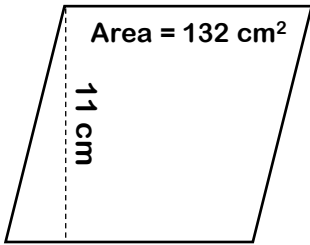
Base:

Height:

Area:

Base:

Height:

Area:

Base:

Height:

Area:

Area = 46 km²

4 km

Area = 0.27 mi²

0.3 miles

Area = 875 mm²

25 mm
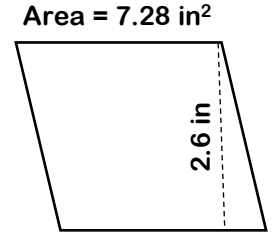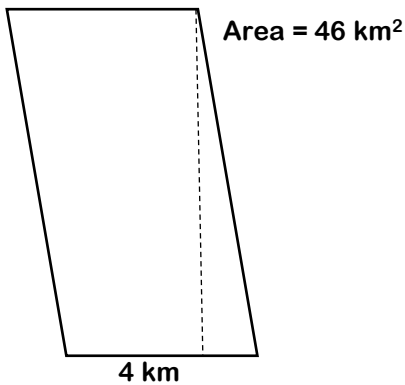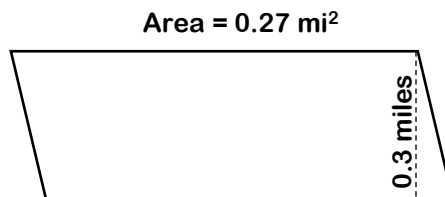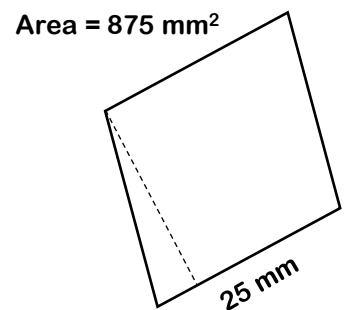
Base:

Height:

Area:

Base:

Height:

Area:

Base:

Height:

Area:

Area of Triangles
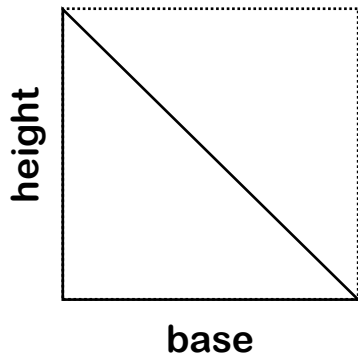
Think of the area of a triangle as *half* the area of a square or rectangle.

height

base

Area for a triangle is one-half of the two sides multiplied together.

The area is ½ times the *base* times the *height* of a triangle.

Area = ½ b x h

Area = ½ (4 x 3) = 6 in²

height = 3 in

base = 4 in

Area = ½ (4 x 3) = 6 in²

height = 3 in

base = 4 in
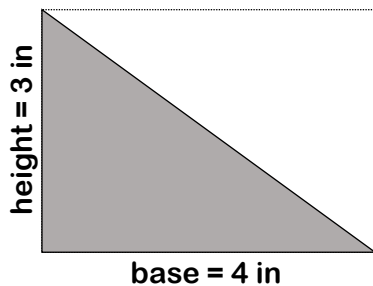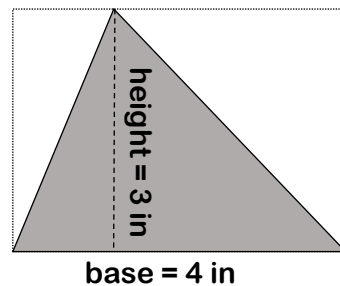
Find the dimensions and area of each triangle:

	4 m / 2 m / 4.5 m	15 in / 12 in / 10 in
Base:	**Base:**	**Base:**
Height:	**Height:**	**Height:**
Area:	**Area:**	**Area:**
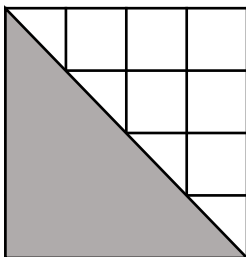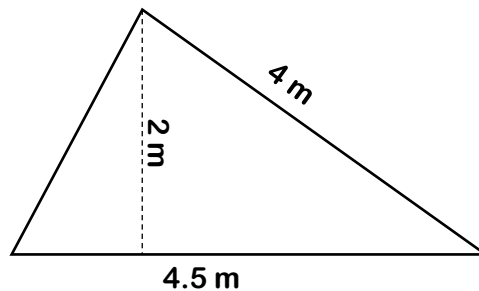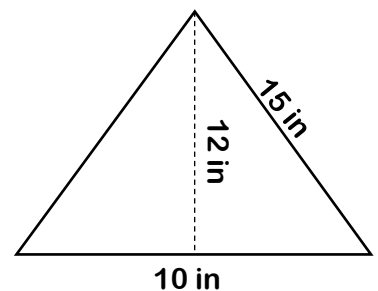
Area of Triangles

Find the area of each shape using Area = ½ base x height
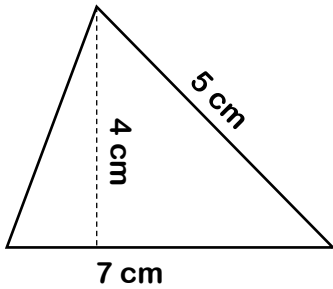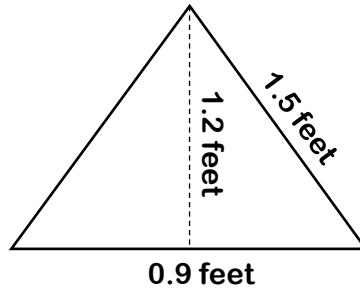
Don't forget units in your numbers!

5 cm · **4 cm** · **7 cm**

1.2 feet · **1.5 feet** · **0.9 feet**

7.5 in · **4 in** · **9 in**

Base:

Height:

Area:

Base:

Height:

Area:

Base:

Height:

Area:

0.57 km · **0.4 km** · **0.76 km**

2.7 miles · **1.1 miles** · **3.1 miles**

1100 mm · **900 mm** · **300 mm**
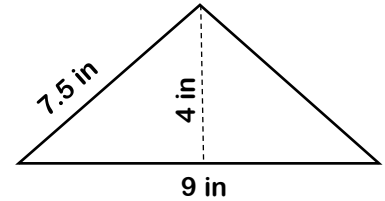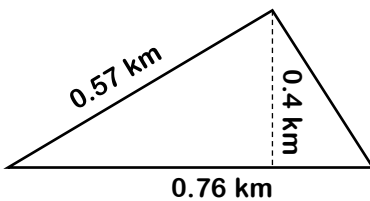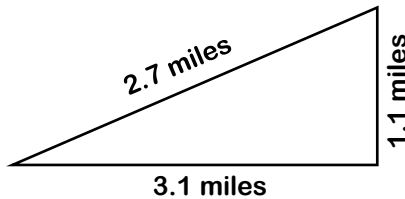
Base:

Height:

Area:
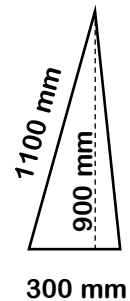
Base:

Height:

Area:

Base:

Height:

Area:

Area of Triangles

Using Area = ½ Base x Height, find the missing dimension(s) for each shape.

Don't forget units in your numbers!

Area = 16 cm²

8 cm

Base:

Height:

Area:

Area = 7 ft²

2 feet
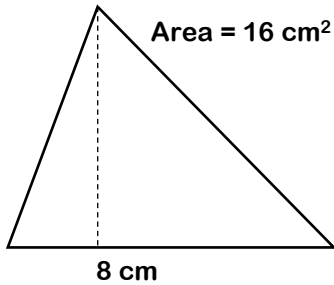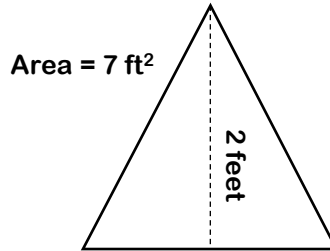
Base:

Height:

Area:

Area = 200 mm²

100 mm

Base:

Height:

Area:

Area = 9 mi²

6 miles

Base:

Height:

Area:

Area = 1,026 km²

76 km
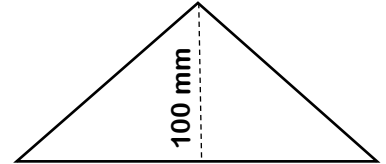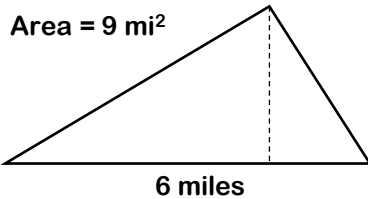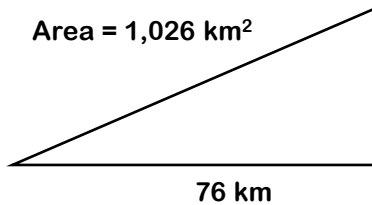
Base:

Height:

Area:

Area = 13.5 yd²

9 yards
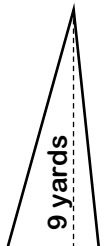
Base:

Height:

Area:

Area of Trapezoids

A trapezoid is one or two triangles and a rectangle (or square) combined.

Three figures are shown below. Find the area of each.

5 cm   4 cm   **3 cm**   **4 cm**   **6 cm**   4 cm   5 cm   **3 cm**

Area:

length of base: b_1

height: h

length of base: b_2
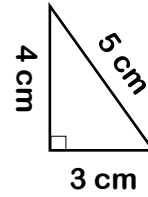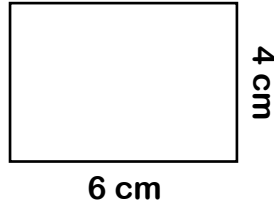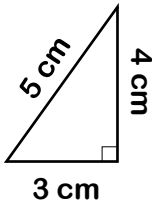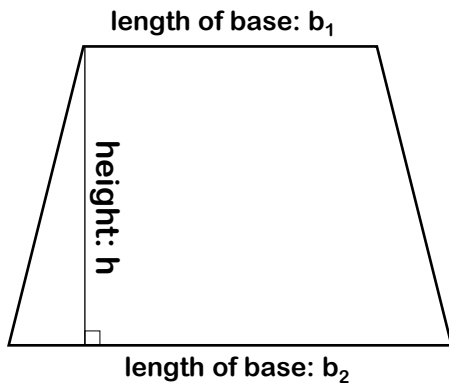
Rectangle Area = b x h

Triangle Area = ½ b x h

To find the area of a trapezoid, we combine the two areas above into this:

Trapezoid Area = ½ (b_1 + b_2) h

Remember! Units for area are *squared* like this: cm^2 or in^2

Find the dimensions and area of each trapezoid:

b_1 = 9 cm   h = 3 cm b_2 = 13 cm

b_1 = 14 cm   h = 7 in b_2 = 21 cm

b_1 = 0.5 mm   h = 0.5 mm b_2 = 1.2 mm

Area: **Area:**   **Area:**

Area of Trapezoids: A = ½ (b$_1$ + b$_2$) h

Use your ruler to measure each shape and find the area in cm^2.

Area:	**Area:**	**Area:**

Draw one trapezoid in each space below. Measure the sides you need and calculate the area.
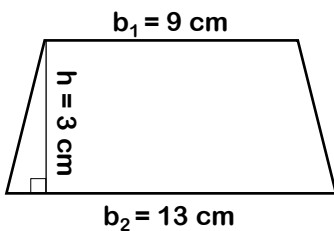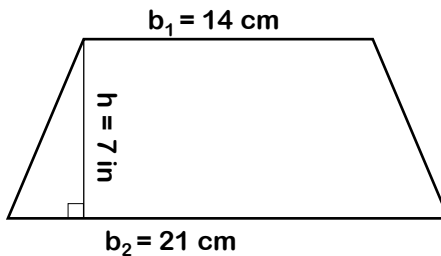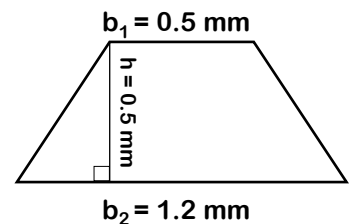
Area:	**Area:**	**Area:**

Area of Trapezoids: $A = \frac{1}{2}(b_1 + b_2)\,h$

These trapezoids are missing a dimension! Can you figure out each one?
Hint: You may have to re-arrange the trapezoid area equation to use it to find the missing side.

b_1 = 16 cm

b_2 = 26 cm

Area = 252 cm^2
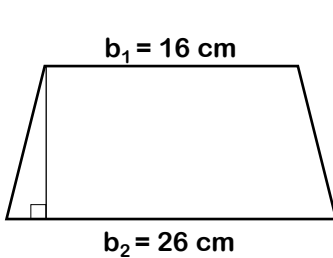
Base 1:

Base 2:

Height:

Formula:

Area:

b_1 = 100 miles

b_2 = 160 miles

Area = 5,200 mi^2

h = 12 mm

b_2 = 22 mm

Area = 204 mm^2

Area = 56 ft^2

h = 7 feet

b_2 = 8 feet

b_1 = 5 cm

b_2 = 15 cm

Area = 20 cm^2

b_1 = 4 mm

b_2 = 7 mm

Area = 22 mm^2
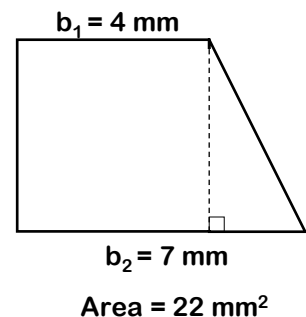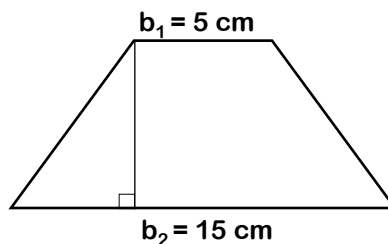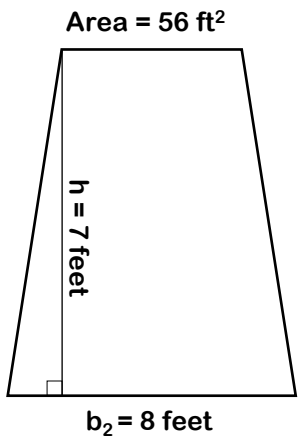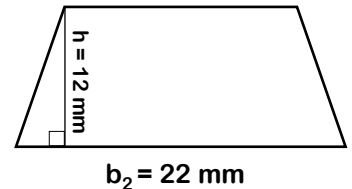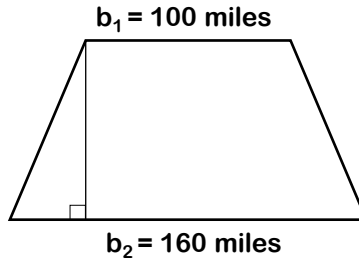
Area of Composite Figures

A *composite* figure is made up of two or more shapes.

How would you decompose the figures below?
Discuss and label the different ways could break each apart into familiar shapes.
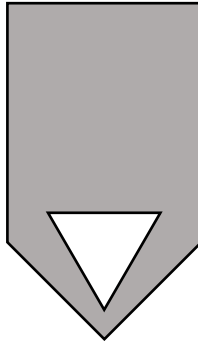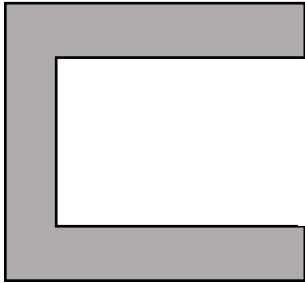
To find the area of a composite shape, first break the figure into familiar shapes,
then add or subtract the area for each shape as appropriate.

9 feet

5 feet

8 feet

4 cm

4 cm

4 cm

8 cm

4 cm

8 cm
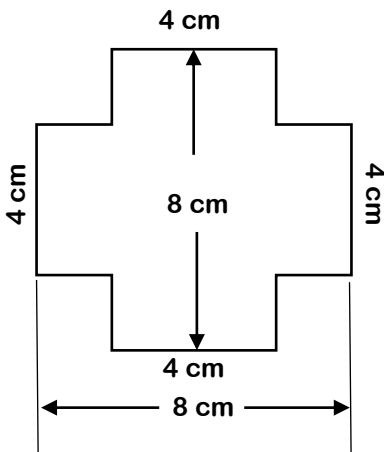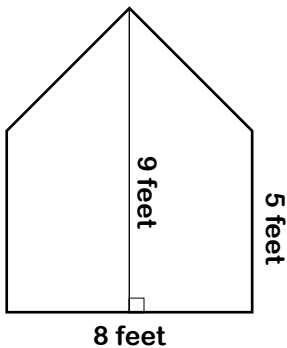
Area of Composite Figures

Find the area of the shaded regions by decomposing it into familiar shapes.

10 in

10 in

Windows are each 40mm x 20 mm

180 mm

100 mm

160 mm

9 feet

8 feet

4 feet

12 feet
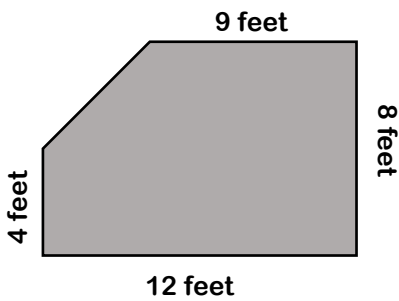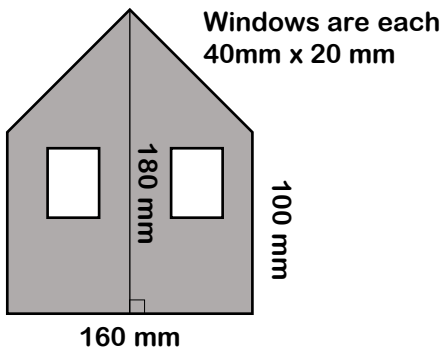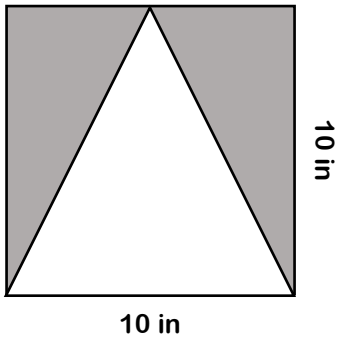
Area of Composite Figures
Calculate the area of the shaded regions below.
Use your ruler to measure any information you need.

Measure in inches

Measure in cm

Measure in mm
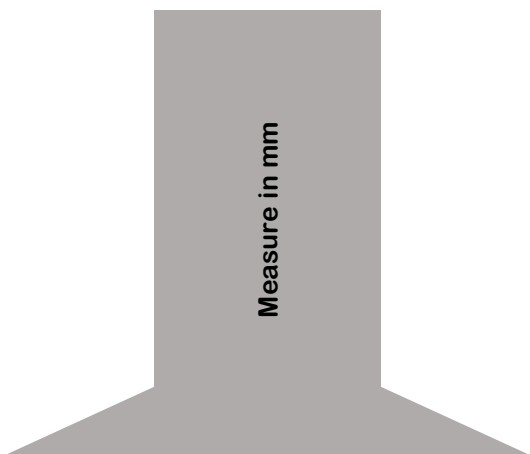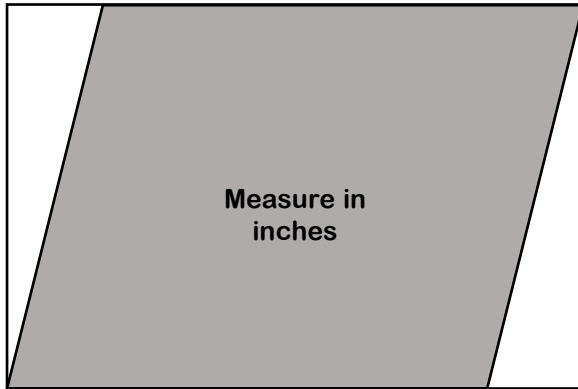
Area of Composite Figures
Solve the word problems by first drawing the shapes.

Tony is placing new 12" square tiles around his 7 ft x 4' pool. Tiles cost $2.25 each. How much is this project going to cost? (Hint: Don't forget the corners!)

Sophia is building a new deck for her home. The deck is a trapezoid shape with parallel sides measuring 20 feet and 30 feet, and distance between the parallel sides is 12 feet. If wood is $3.40 per sq ft, how much will it cost to build the deck?
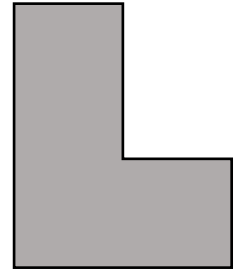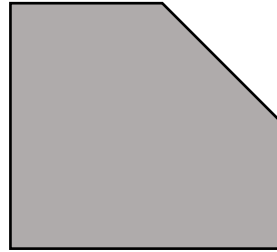
Area of Composite Figures
Solve the word problems by first drawing the shapes.

Arthur lives in a home with ten foot ceilings. He is going to paint two 12-foot long walls with two coats of paint. One wall has a 3 ft x 5 ft window. One gallon of paint covers 400 sq ft. How much paint should he buy?

Trent is laying new tile in his kitchen. The kitchen measures 14 feet by 20 feet. The kitchen has cabinets that extend 2 feet into the room sitting along the 14 foot side. There's also a 6 x 8 foot center island that he is going to work around. The tiles are 12 inches square for $5.50. How much tile does he need and how much will it cost?

Area of Circles
Circles are round shapes without any corners or line segments.
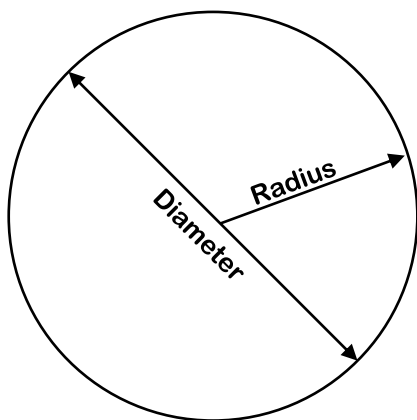
The Radius (r)
This is measured *from* the center of a circle to any point *on* the circle.

Area = π r²

The Diameter (d)
Any straight line passing through the *center* of the circle with endpoints *on* the circle.

Area = ¼ π d²

π = 3.14159…

π: (*pi*, pronounced *"pie"*) is a number a little larger than 3.

π is the number we get when we divide the circumference by the diameter.

We will use π = 3.14 for our calculations.

The Circumference (C)
This is the distance measured *around* the entire circle.

C = 2 π r = π d

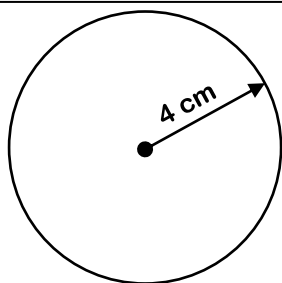### Find the dimensions and area of each circle:

r = 4 cm

d = 2 x 4 cm = 8 cm

C = 2 π r = 2 (3.14)(4 cm)
        = 25.1 cm

Area = π r² = (3.14)(4 cm)²
        = 50.3 cm²
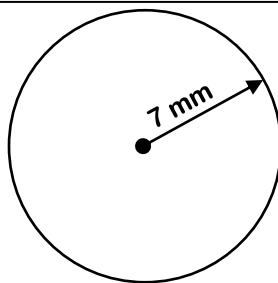
r:

d:

C:

Area:
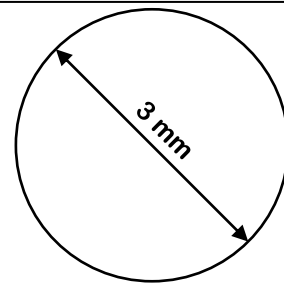
r:

d:

C:

Area:

Area of Circles

Circumference = 2 π r   **Area = π r^2 = ¼ π d^2**   **Use: π = 3.14**

A wooden horse on a carousel travels in a circle. The carousel is 20 feet in diameter. How far does the horse travel each time it completes a circle?

What is both the radius and diameter of the carousel?

The owner needs to install a new floor in the carousel. The carousel has a 4 foot hole in the center for the machinery that turns the carousel. How much wood does he need, in square feet?

Area of Circles

Circumference = 2 π r   Area = π r² = ¼ π d²   Use: π = 3.14

An electric model train travels in a circular loop of track made from 4-foot radius arcs. How far does the train travel after 10 laps around the track?
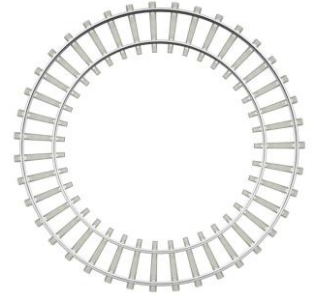
What is both the radius and diameter of the track?

Approximately how many laps does the train need to do to travel 500 meters?
1 meter = 3.28 feet

Area of Circles

Circumference = 2 π r   **Area = π r^2 = ¼ π d^2**   **Use: π = 3.14**

A farmer's rotating irrigation sprinkler setup covers an area of 2800m^2. About how big is the circle that it covers? (Calculate both radius and diameter.)

A horse trainer wants to create a 50-foot diameter bullpen (a round fenced-in enclosure) for training and lunging horses. How many fences does she need if they come in 7 foot sections?

Practice Test

Use your ruler (use cm) to determine the dimensions, and then calculate the perimeter and area of each shape below. Round to the nearest tenth of a cm.

Diameter:

Radius:

Circumference:

Area:

Now draw a similar circle that is one quarter the size of the original.

Length:

Width:

Perimeter:

Area:

Now draw a similar rectangle that is three times the size of the original.

Length:

Height:

Perimeter:

Area:

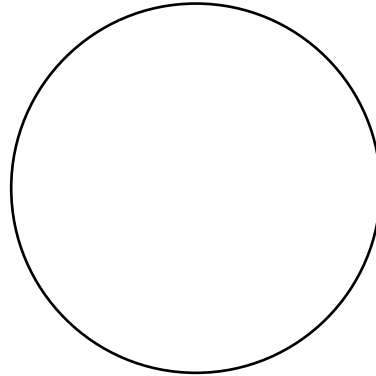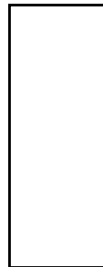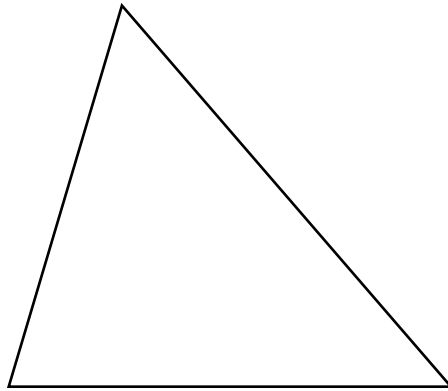**Now draw a congruent parallelogram.**

Practice Test

Use your ruler (use cm) to determine the dimensions, and then calculate the perimeter and area of each shape below. Round to the nearest tenth of a cm.

Base:

Height:

Perimeter:

Area:

Now draw a similar triangle that is only half the size of the original.

Base 1:

Height:

Base 2:

Area:

Now draw a similar trapezoid *inside* the original of any scale factor.

Length:

Width:

Radius

Perimeter:

Area: